EXTENDED SVD FLATNESS CONTROL

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ABSTRACT

Cold rolling mills sometimes do not seem able to control flatness as well as expected, taking into account the number of actuators they are equipped with. This paper, however, presents a systematic approach to the question of how to achieve the accurate flatness control that you would expect to get in a well-actuated cluster mill. The key to understanding the restrictions and difficulties is singular value decomposition (SVD) of the matrix that describes the flatness responses for all actuators. This decomposition clarifies which flatness error shapes are easy to counteract, in the sense that they require little actuator movement, and which error shapes are difficult or even impossible to reduce, since they would require very large combined actuator movements. Having sorted this out, a control strategy that uses this knowledge is presented. Experience and results are provided from its successful use in practice.

KEYWORDS

Flatness control; Singular value decomposition; Cold rolling; Cluster mill; Robustness; Performance;

INTRODUCTION

Flatness control in cold-rolling is a multivariable control problem. Abstractly speaking, you can view flatness errors and actuator movements as occurring in spaces of high dimension. Different shapes of the flatness error correspond to different directions in the space of flatness errors, and the dimension of this space equals the number of sensors. Different combinations of actuator movement correspond to different directions in the space of actuator movements, and the dimension of this space equals the number of actuators.

A combined movement of actuators in the actuator space will influence the errors in a certain direction in the error space and this with a certain gain. This gain will vary a lot with the direction of the movement. Singular value decomposition (SVD) is the systematic way to get insight into this. It orders the influence into orthogonal directions in the spaces according to the gains, from highest gain to lowest, often ending with some directions having zero gain. Use of SVD in flatness control has previously been described in [1], [2], and [3]. You sometimes see flatness errors being described using a basis of orthogonal polynomials. The SVD provides another orthogonal basis, with the additional advantage that for each error basis vector you also get the corresponding actuator basis vector and the gain.

The largest singular value and the corresponding basis vectors tell which shape of flatness error is the easiest to eliminate, in the sense that it takes the least actuator movements. It also tells how the required combined actuator movement looks. This is continued for gradually more difficult-to-reduce flatness error shapes, corresponding to directions in error space that are orthogonal to the ones already treated. If the smallest singular values are zero, then the corresponding actuator directions form a null space, meaning that movements of actuators in this space cancel out the flatness effect of each other. In other words, when there is a null space the flatness effect obtained with one actuator can alternatively be obtained by a certain combined movement of the other actuators.

It often turns out that a mill with ten actuators or more can still not control more than around five directions in practice, since the remaining ones would require too large actuator movements. The problem is not only the restricted actuator ranges. For a direction related to a very small singular value, the responses to the large movements are supposed to cancel in all directions except the intended one. But to achieve this precise cancellation, the model would have to be unrealistically perfect. This does not mean that the extra actuators are useless. It just exposes the control challenge. The control solution should use all actuators and all degrees of freedom, but not nervously spend large movements on chasing errors that are too hard to counteract in practice. In contrast to the solutions presented in [1] and [2], all degrees of freedom are still available for control with the solution presented in this paper.

The paper presents the mathematical background in section 1. The control solution and some of its properties are presented in section 2. A discussion of the tuning of this solution is given in section 3. Practical experience is exemplified in section 4, and some concluding remarks are provided in section 5.

1. THE MILL MATRIX AND ITS SINGULAR VALUE DECOMPOSITION

The influence of the actuators on the flatness can be described by a matrix which we call the mill matrix. Its columns are formed by the steady state flatness responses from all actuators, one column per actuator. Each flatness response is a column vector with one element per measurement roll sensor position. So the dimension of the mill matrix is (number of sensor positions) times (number of actuators). We denote it by G_m and spell out the steady state relation as

$$\Delta e = -G_m \Delta u_a \tag{1}$$

Here Δu_a is a vector describing a change in actuator positions from a previous steady state, and Δe is the resulting vector of steady state change in flatness measurement. The minus sign is just a choice of convention.

Flatness control is a multivariable control problem, and in multivariable process control directionality plays an important role. You can talk about high gain directions and low gain directions, where the former are easy to control and the latter are difficult. It is often hard to get a good enough model to be able to control the low gain directions, and you might need to give up control of them. To find out what high and low gain directions we have in our flatness control case, we make a singular value decomposition of the mill matrix. This is the mathematic expression for it:

$$G_m = U\Sigma V^T = \begin{bmatrix} U_1 & U_2 & U_3 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 \Sigma_1 V_1^T \tag{2}$$

In this expression, the upper part of Σ is a diagonal matrix holding the singular values $\{\sigma_1, \sigma_2, \ldots\}$ in the diagonal, and they are ordered from largest to smallest, and all are positive or zero. The lower part of Σ is formed by as many zero rows as there are more sensor positions than actuators. The matrix U is orthonormal and its columns form a basis for the space of flatness errors. In the same way the matrix V is orthonormal and its columns form a basis for the space of actuators. We have assumed that there are more sensor positions than actuators. The singular values are the gains from an actuator movement along a column of V to the response in flatness, which actually occurs along the corresponding column of U. In the second equality in (2), we make a partitioning of the matrices according to high gains and low gains (and those which are zero due to fewer actuators than sensor positions). In the last equality in (2), it has been assumed that all singular values in Σ_2 are zero, or small enough in comparison with the larger singular values to be approximated to zero.

Now, we will try to explain in simple words what the singular value decomposition (SVD in short) can tell about the properties of the mill matrix and thus about the multivariable flatness control problem.

Suppose we stand in the high dimensional actuator space and want to take a step of size one, wondering what direction to choose in order to get the largest possible flatness influence. The answer is: Choose the direction given by the first column of V. The size of your flatness response to this particular combined movement of actuators will be σ_1, the largest singular value, and the shape of the steady state response will be given by the first column of U. The singular value is the gain, and this was the high gain direction. If you then look for the highest gain among remaining actuator directions orthogonal to that first one, you will find the second singular value. It is the gain from actuator movement along the second column of V, which gives flatness response along the second column of U. We can continue like this towards lower and lower singular values (gains), until we reach those which can be approximated to zero. We have then found also the low gain directions.

What is it then that makes high gain directions easy to control and low gain directions hard? In the high gain direction, small actuator movements will be enough, and you may disregard interference with other directions. Control in a low gain direction is troublesome for at least two reasons: 1) it will require large actuator movements which may cause rate saturation and even absolute saturation, which both have negative influence on performance, 2) the effect of the large actuator movements are supposed to cancel each other in the higher gain directions, and this puts hard demands on model accuracy, often harder than possible to fulfill.

The number of non-zero singular values is what is called the rank of the mill matrix. We may define the ‘practical rank’ as the number of singular values that we consider to be high enough gain to be addressed by the flatness control. The singular values that are smaller define a (practical) null space of the mill matrix. Actuator movements in the null space have no (or insignificant) influence on the flatness. The partitioning in (2) is such that the practical rank is given by the size of Σ_1, and Σ_2 holds the singular values that are considered too small. The null space is spanned by the columns of V_2. Another aspect of this is that flatness errors that can be expressed as linear combinations of the columns in U_2 and U_3 cannot be counteracted at all. There are no combinations of actuator movements that influence errors of those shapes. All flatness errors that can be counteracted can be expressed as linear combinations of the columns of U_1.

A recording from a mill is shown in Figure 1, where the initial flatness and the final flatness are equal, but with quite different actuator positions. It has been ascertained that this was not due to a changed disturbance situation. This illustrates the fact that the mill matrix for this mill has a null space. The actuator movement from the initial positions to the final ones did only cause a transient change of the flatness, but no change of steady state flatness, so the same flatness effect from the actuators was obtained with both final and initial positions. Further, towards the end the actuators still move without influencing the flatness, so this movement also takes place in the null space. This mill has eleven actuators and the mill matrix has rank eight, so the null space has dimension three. The practical rank is probably rather around five, leaving a practical null space of dimension around six.

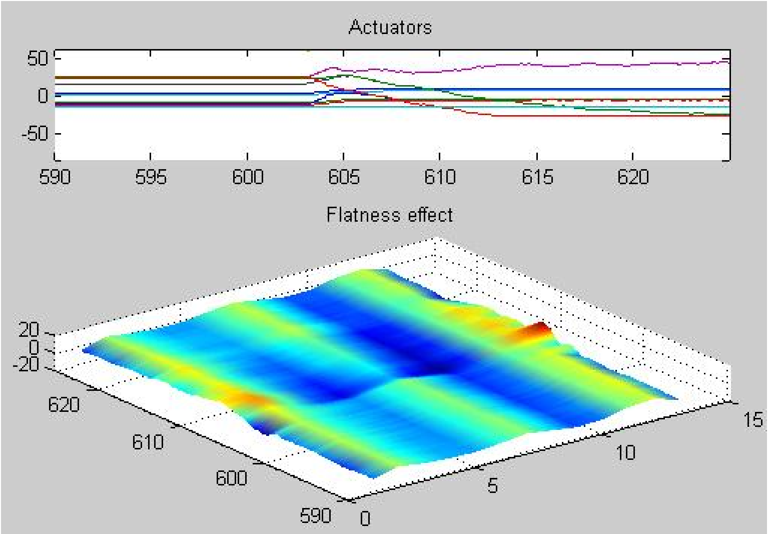

Figure 1. An actual recording from a mill, illustrating that the same flatness influence is obtained with quite different actuator positions. The difference in initial and final positions represents a movement in the null space.

In Figure 2 and Figure 3, an example with seven actuators is used to illustrate the singular value decomposition of a mill matrix. This example has five crown actuators and two side shifts. This example does in fact not have an extremely high ratio between largest and smallest singular values. This ratio is called the condition number of the mill matrix. The largest singular value is in this case σ_1=3.6 and the smallest σ_7=0.086, giving a condition number of 42, which is not too bad. But if there is some uncertainty in the response functions, one might still want to treat actuator movements along the last SVD direction as having no effect (i. e. as being in the null space), since the effect of these movements will be small and uncertain. Total elimination of errors shaped according to the last SVD direction would in any case require very large actuator movements, thereby easily causing them to reach their constraints. It is therefore wise to give up at least total elimination of such errors.

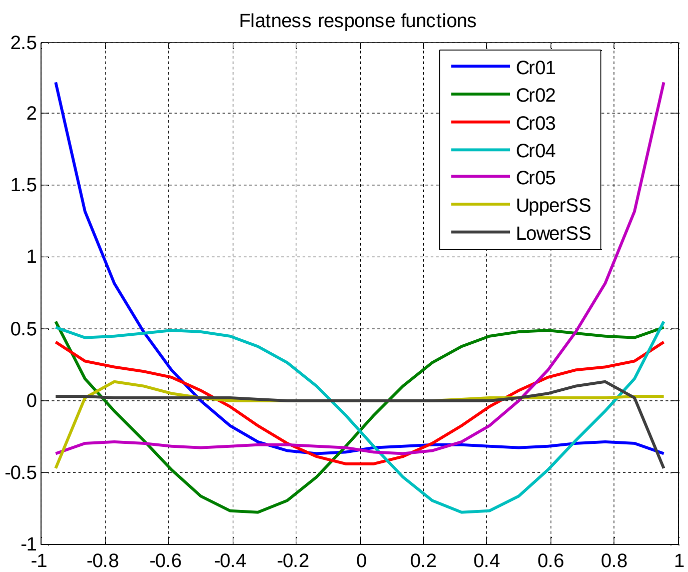

Figure 2. Steady state flatness response function for each of the actuators

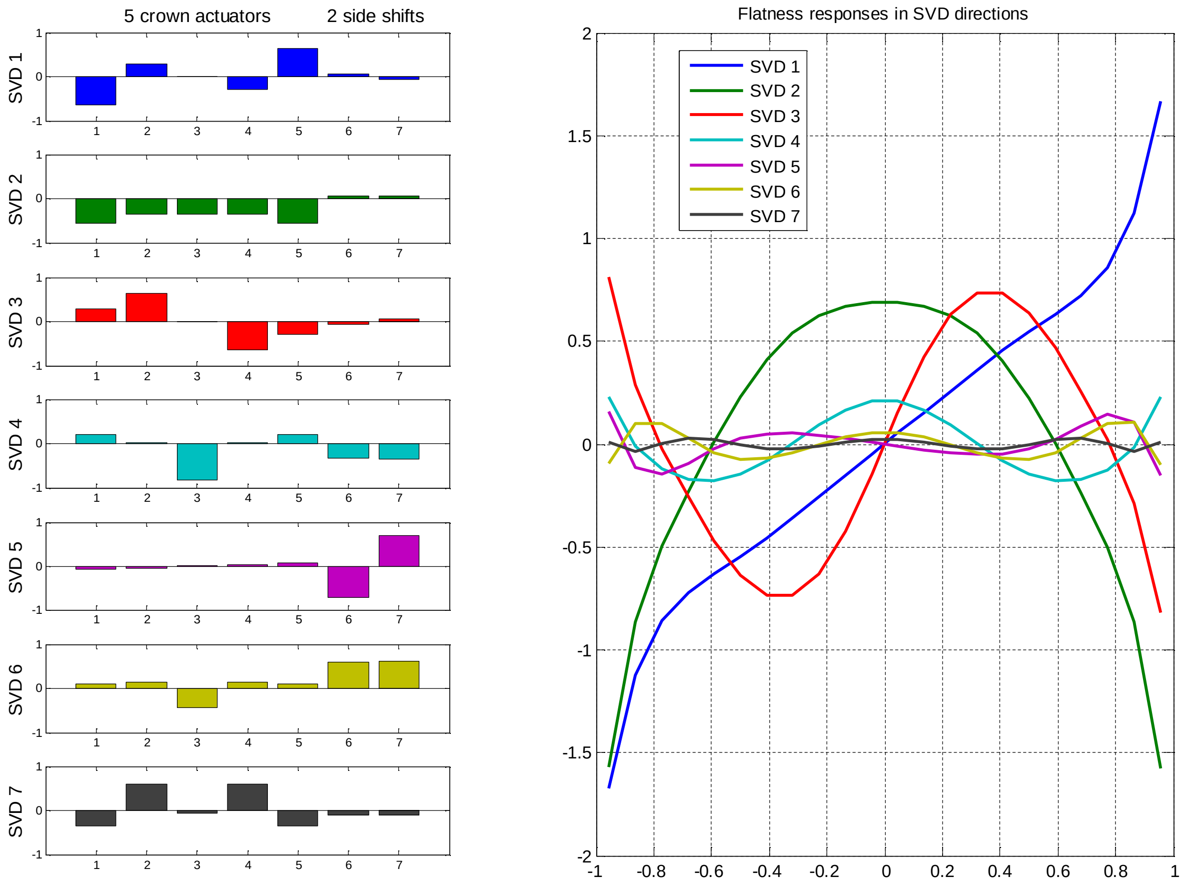

Figure 3. Flatness responses along singular value directions. The left hand plot shows combined actuator movements that correspond to the respective singular value, and the right hand plot shows the flatness response for each of those combined actuator movements. Largest response is obtained when actuators are moved along the first SVD direction and smallest response when moved along the last.

2. CONTROL SOLUTION

One common control solution uses a parameterized flatness error obtained by minimizing a quadratic criterion while honoring actuator constraints. Based on SVD of the mill matrix, this solution can be extended to include weights on directions associated with small singular values, thereby providing the desired control performance. This "extended SVD control" solution uses all actuators available. It can also move them in the null space, when needed due to constraints, but will not cause unnecessary movement there. The standard control solution is illustrated in Figure 4.

This standard control solution minimizes a criterion to find a lower dimension parameterization e_p of the flatness error e. Each element of the parameterized error vector e_p is fed to one controller, for example a PI controller, and the output of each controller is fed to one of the actuators. In its basic original form the criterion to minimize could be expressed as

$$W(e_p(t), e(t)) = \left\| G_m e_p(t) - e(t) \right\|^2 \tag{3}$$

The criterion (3) is to be minimized while taking constraints into account. Those constraints are the actuator constraints, including their rate and range constraints and for example constraints on differences between adjacent crown actuators. Denoting the vector of actuator positions at time t by

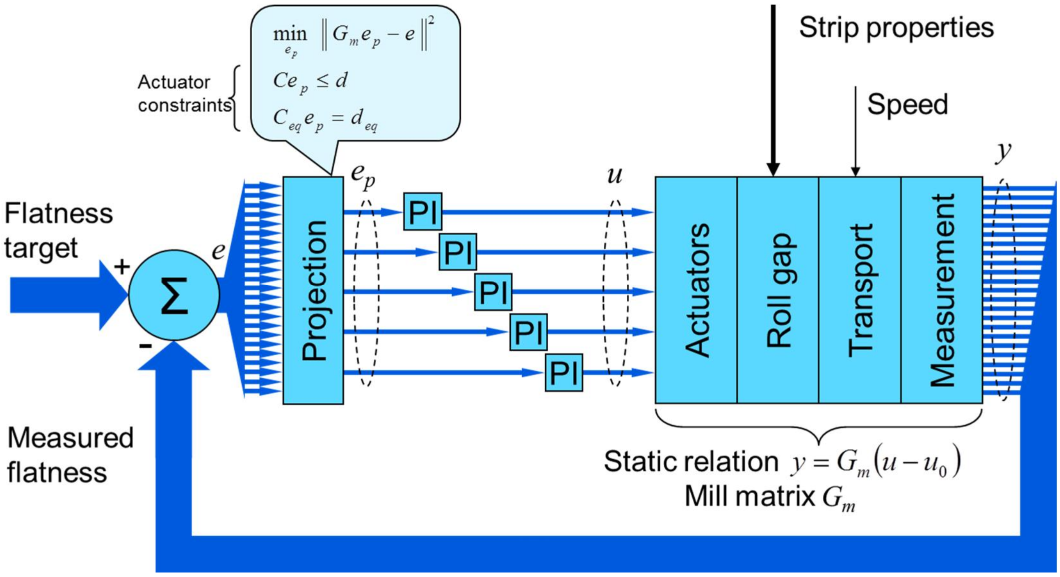

Figure 4. Block scheme showing the structure of a standard control solution

$u(t)$, this $u(t)$ which is subject to constraints will depend on the parameterized error $e_p(t)$, the previous positions $u(t\text{-}1)$ and the controller states. In cases when the constraints do not become active, the solution to this simple minimization is given by the pseudo-inverse G_m^+ of the mill matrix G_m. Thus, for a case with non-singular mill matrix and no active constraints we get the parameterized error by the projection

$$e_p(t) = G_m^+ e(t) = \left(G_m^T G_m\right)^{-1} G_m^T e(t) \tag{4}$$

This expression will however get very sensitive, if the mill matrix has any singular value that is small in relation to the largest one. The inverse $\left(G_m^T G_m\right)^{-1}$ will not even exist, if any singular value is zero. To make this solution, relying on minimization of a criterion like (3), practically usable for cases with a singular or near singular mill matrix, you need to either add some kind of regularization to the criterion, or make the mill matrix better conditioned for example by gathering a number of actuators into fewer virtual actuators. The latter approach will however remove some degrees of freedom that the mill could have benefitted from.

Based on the singular value decomposition, we can introduce a systematic regularization that enables the full use of all degrees of freedom while still avoiding the problems associated with a (near) singular mill matrix. A straightforward version of the extended criterion is

$$W\left(e_p(t), e(t), u(t)\right) = \left\|G_m e_p(t) - e(t)\right\|^2 + e_p(t)^T V Q_e V^T e_p(t) + u(t)^T V Q_u V^T u(t) \tag{5}$$

It is still to be minimized while taking constraints into account, just as with the original criterion (3). The idea here is that Q_e and Q_u are chosen as diagonal matrices. The entries in the diagonals imply certain weights in the criterion for parameterization of the error and the actuator positions, respectively, along the directions sorted according to the singular values (or in other words sorted according to the gains). So, for example, if we consider the gains to be too low from the fifth singular value and on, then we should choose the fifth and further diagonal elements of both Q_e and Q_u high enough. The first four diagonal elements in these two matrices may in this example be set to zero, meaning that no extra weight is applied to parameterizations that give actuator movements in the first four directions. The choice of Q_e will mainly influence the transient behavior, while the choice of Q_u will mainly influence the steady state behavior.

Assuming that, in the criterion (5), the non-zero weights in Q_e and Q_u correspond to the null space, then movement in the null space will be avoided as long as no actuator gets saturated. But when any actuator hits a constraint, the control solution will use other actuators to accomplish – at least partially – what the saturated one could not. This is possible since movement in the null space is allowed. Such movement is still avoided when not needed, since there is a penalty in the criterion for using it.

3. TUNING PARAMETERS AND TUNING TOOL

It is apparent from section 2 that the two matrices Q_e and Q_u are important for the control behavior in the extended SVD control solution. They are, however, not really the kind of intuitive tuning parameters that a commissioning engineer would require. What is needed in addition is a tuning tool that allows the tuning to be performed on a higher level, providing easily grasped tuning knobs and clear presentations of what the expected results will be.

In tuning, which is made at commissioning, one consideration is the trade-off between robustness and nominal performance. One measure of robustness is the peak of the sensitivity function. For process control it is often chosen to be between 1.2 and 2.0. Lower values in this range mean higher robustness towards deviations between the model used in tuning and the actual plant behavior, but also slower counteraction of disturbances and therefore lower nominal transient performance. Only if you have a model that you trust very well, you would tune the controller to get sensitivity peak values in the upper part of the suggested range. If you expect a need for very high robustness, due to an uncertain model or varying actual behavior, you could very well tune for a sensitivity peak below 1.2 as well. The lower limit for what you can get at all is 1. With a tuning knob for the desired sensitivity peak value, the engineer doing the tuning will have good influence over the robustness to be obtained. This covers the multivariable aspect of robustness, as we consider the maximum singular value of the multivariable sensitivity function.

The sensitivity peak specification is an indirect specification of transient behavior, and it eventually leads to values of the diagonal of Q_e, found to give the desired peak value. The user also has the freedom to select how many directions should be acted on with full force. Those will have zeros in the corresponding diagonal position in Q_e, and the remaining diagonal elements will be used to get the specified sensitivity peak value.

As a sanity check regarding multivariable behavior, it is also made sure that there is not too much cross talk interference between different directions during transients. The user can specify an allowed percentage for this cross talk.

The tuning of the individual controllers (one per actuator) will actually also be part of the tuning of transient behavior. So beside some elements of Q_e, also a parameter related to the settling time for the individual control loops will be found automatically to get the desired sensitivity peak, and cross talk below limits.

An important part of any tuning tool is clear presentation of expected control performance. The transient behavior can here be studied for a number of different disturbances, also with a validation model that may differ from the nominal one used in the tuning. This way one may check both the nominal performance and the robustness. In addition to that, the actually obtained sensitivity peak value is presented, as well as the closed loop time constant for the individual loops.

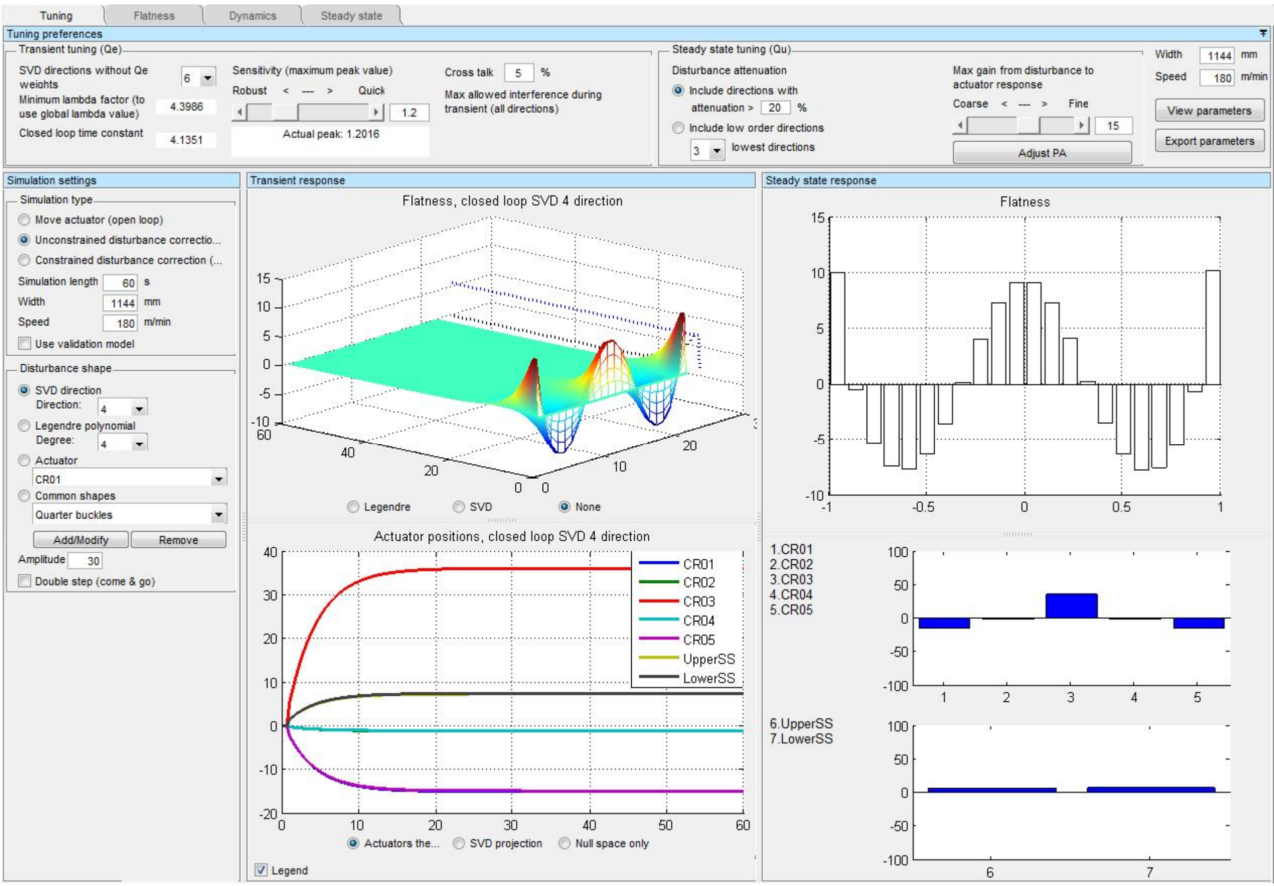

Figure 5. A tuning tool view. On top there are tuning inputs and some resulting performance measures. To the left there are simulation choices, in this case selecting a disturbance along the fourth SVD direction applied in closed loop with the nominal model and ignoring actuator constraints. The graphs are divided as follows: The upper graphs show the flatness response and the lower graphs show the actuator positions. The graphs to the left show the transient response while the graphs to the right shows the result at the end of the simulation. In the upper right graph only the initial errors are visible (white bars), since the final errors are zero in this simulation.

An example view from the tuning tool can be seen in Figure 5. In this view, the evolution of the flatness error can be viewed in a 3D plot (like the figure), or expressed with either the SVD basis or a polynomial basis. The actuator movements can be viewed as is (like the figure) or expressed with the SVD basis, and one can also choose to view their possible movement in the null space.

The steady state behavior with extended SVD control is mainly determined by the choice of Q_u. The concern of the tuning engineer is to avoid too high closed loop steady state gain from disturbance to actuators, since that would too easily cause actuator saturation. For directions corresponding to large singular values of the mill matrix (high gain directions) this is no problem, but it may be for low gain directions. So the Q_u diagonal elements corresponding to large singular values should be zero and the rest should be found based on the setting of a suitable intuitive tuning knob. This knob can be the highest allowed steady state gain from disturbances to actuators. The tool can easily translate that to required values for the Q_u diagonal elements and calculate the resulting steady state attenuation of disturbances. In addition, it is possible to let the engineer choose for how many SVD directions disturbances should be completely eliminated in steady state, provided the mentioned gains do not exceed the specified limit. And it is possible to state that if disturbances in any SVD direction cannot be attenuated more than a certain percentage, control of it should be given up totally. All these settings may influence the finally obtained diagonal elements of Q_u.

To check the result of the steady state tuning a view is available in the tool, as exemplified in Figure 6. In this example, disturbances along the first six SVD directions are totally eliminated in steady state, but since a limit of 15 was specified for the steady state gain from disturbance to actuators, disturbances along the last SVD direction will be attenuated only by 36%. The limit value is normalized with respect to the first SVD direction, which means that a value of 15 allows 15 times more actuator movements than the first SVD direction. This view will help the tuning engineer check the expected behavior and select a suitable steady state tuning.

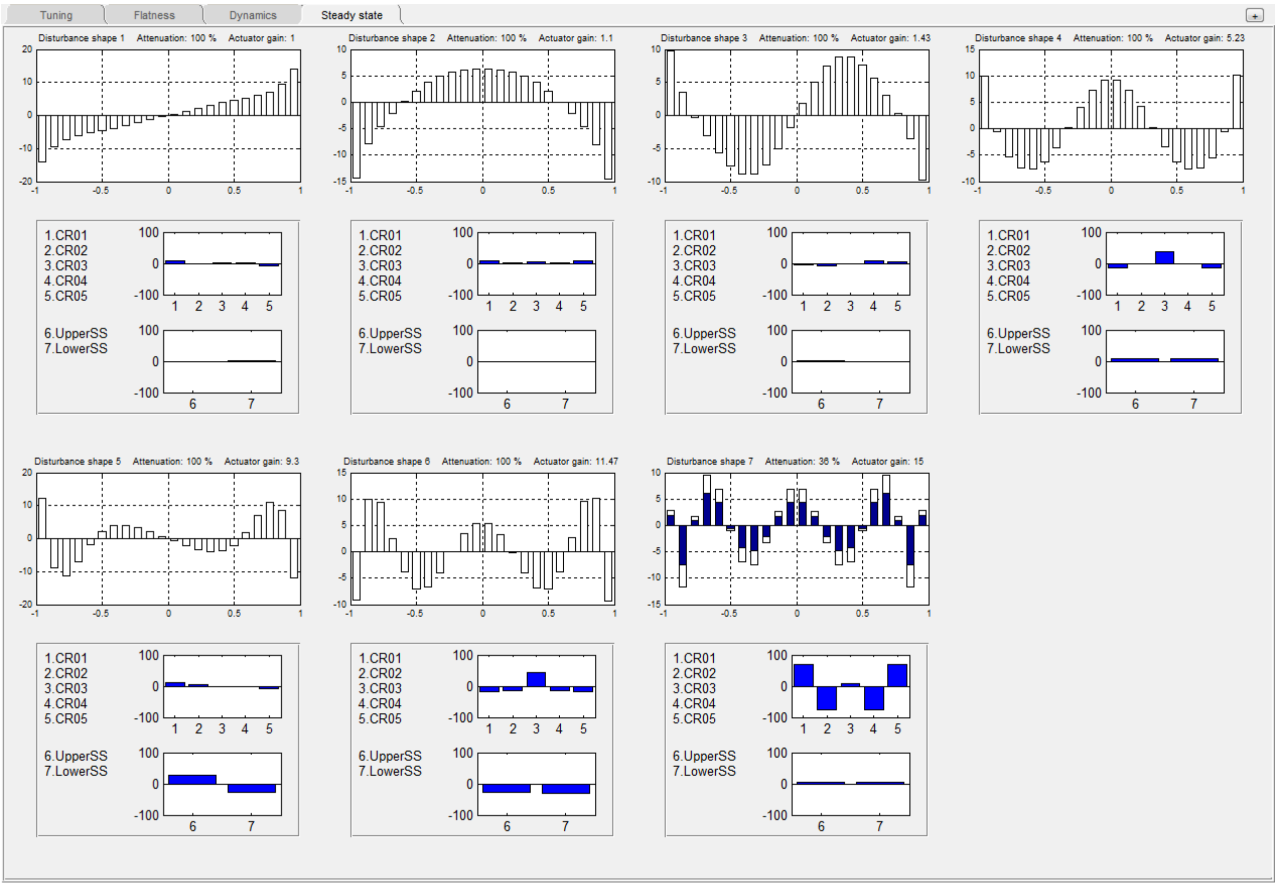

Figure 6. A tuning tool view presenting the steady state responses in closed loop. For each SVD direction, there is one plot showing an initial and final flatness error (white and blue bars respectively), when a disturbance of that shape occurs, and below that the final position of the actuators (blue bars), assuming they were in zero position before the disturbance occurred. Here, each disturbance has the same norm, so the actuator movements required to reach the presented final error can be directly compared.

4. PRACTICAL EXPERIENCE

The presented control solution extended SVD has been commissioned in a cluster mill containing 11 actuators with great results. The mill matrix in this mill has a theoretical rank of eight. The practical rank, however, was considered to be four, leaving a practical null space of dimension seven. The ratio between the largest (first) and sixth singular value was 130, which means that it would require 130 times larger actuator movements to eliminate a disturbance according to the sixth SVD direction in comparison to a disturbance of the same size for the first SVD direction. This is far too high to be practically possible. The corresponding ratios for the fourth and fifth singular values were 22 and 38. Both are plausible depending on the accuracy of the mill matrix. The better the flatness response models are, the more SVD directions can be used in control. To determine the model accuracy, a system identification experiment was performed in the mill where the actuators were excited according to their SVD directions and the corresponding flatness responses were recorded. Analysis showed that the recorded flatness responses and the SVD directions from the nominal mill matrix agreed very well up to the fourth direction. The fifth SVD direction, however, did not the match the expected shape since the actuator movements that were supposed to cancel each other out failed in doing so. Instead of a getting a low gain direction, the result was a quite high gain direction of a completely different shape than expected. Of course, this direction could not be included in control, which resulted in a steady state tuning that included four SVD directions as can be seen in Figure 7.

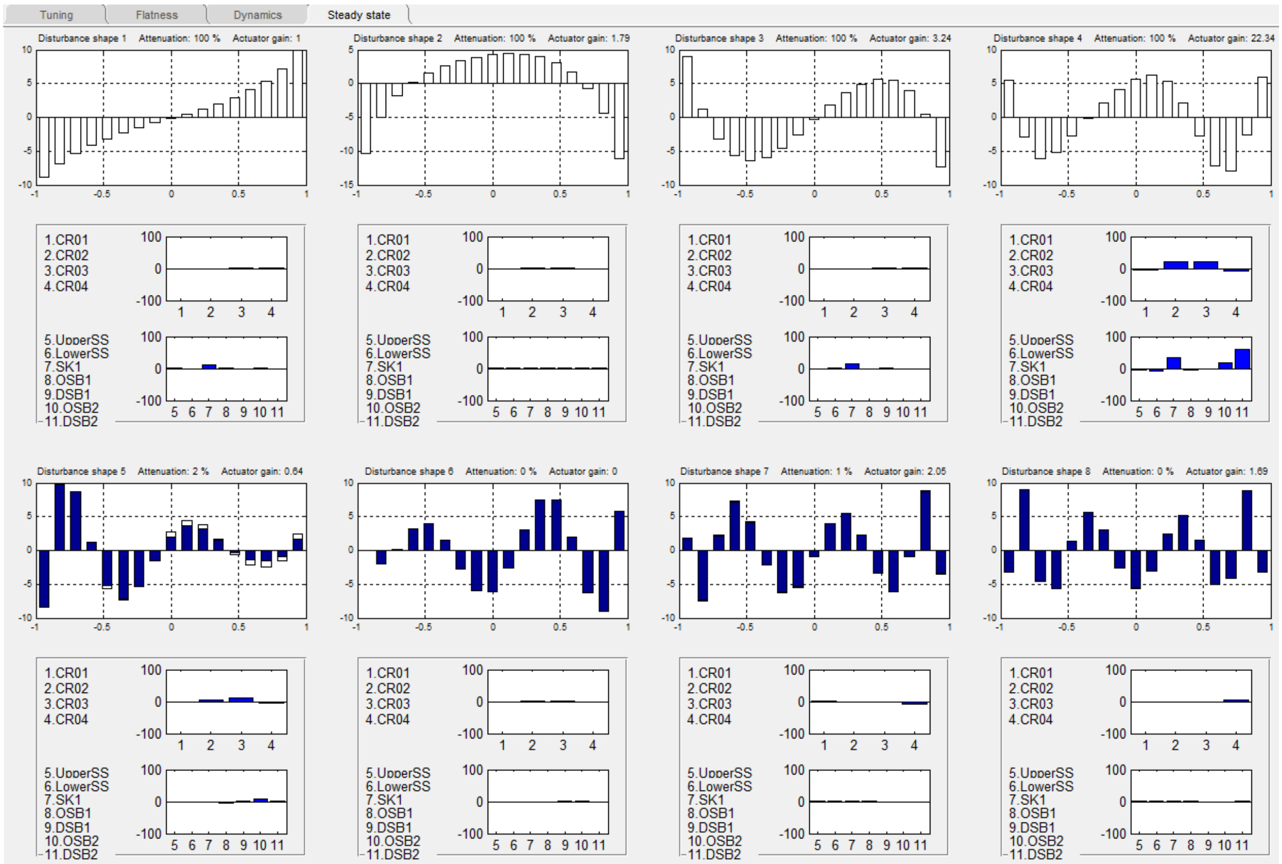

Figure 7. Steady state tuning used in control of the cluster mill. Four SVD directions were used in the control

The former flatness control solution made the mill matrix better conditioned by mapping actuators with similar flatness responses together, thereby reducing the number of control loops. In addition, the crown actuators were not used in the automatic flatness control; they were only used for manual operation by the mill operators. Of course, this control solution has removed valuable degrees of freedom.

In Figure 8 below, a startup of a coil is plotted. The upper graph shows the actuators positions, the middle graph shows the flatness error (flatness target – measured flatness) and the lower graph shows the mean flatness and the used control strategies. A thin blue line at the bottom of the lower graph indicates the former control solution and the thick blue line indicates the use of extended SVD control. As can be seen in both the 3D graph of the flatness error and the graph of the mean flatness, the former control performs poorly due to the fact that several actuators are saturated (as can be seen in the upper graph as horizontal straight lines). When switching to extended SVD control, all actuators could be used in control and the full use of the available degrees of freedom resulted in a significant drop in flatness error and mean flatness. The average flatness was improved from a value of 17 to 4 I-units.

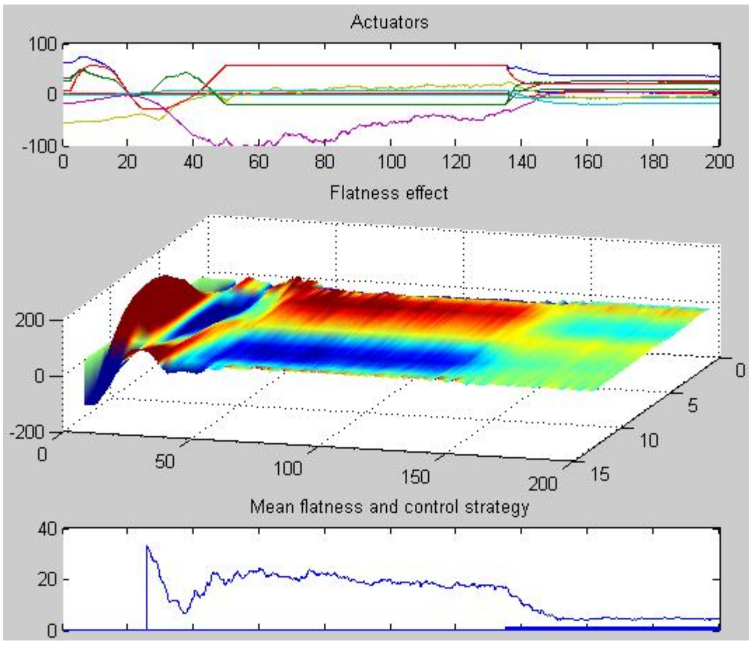

Figure 8. A recording from the startup of a coil, initially using the previous control solution and activating the extended SVD control after roughly two thirds of the recording (thick line in the bottom graph).

Both process engineers and operators at the cluster mill were pleased with the extended SVD control. First of all, all actuators were individually used in the automatic control and thereby using all degrees of freedom. Secondly, the actuators were more centered within their working ranges, which minimized saturation limit situations and thereby could reduce wear and maintenance of the actuators.

5. CONCLUSIONS

We can conclude that the systematic way of treating the mill matrix using singular value decomposition is an efficient way for understanding the multivariable control problem inherent in flatness control for cases with many actuators. The extended SVD control solution that is based on it has also proved to give the desired control accuracy. It retains all degrees of freedom and uses all actuators available. In particular it handles actuator constraints efficiently, since it can use other actuators to provide the same effect, when one actuator has become saturated.

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